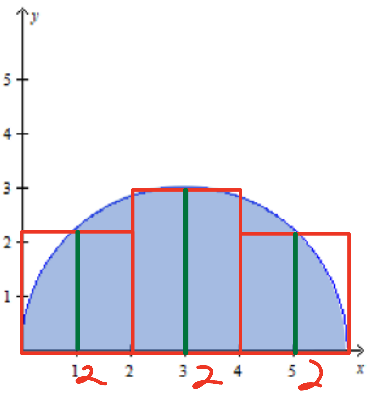
Period 1, Jan 24, 2025

5. The graph of $g(x) = \sqrt{6x - x^2}$ is shown below.

X	$Y = \sqrt{6x - x^2}$
1	$\sqrt{5} = \sqrt{6(1) - 1^2} = \sqrt{5}$
3	$3 = \sqrt{6(3) - 3^2} = \sqrt{9}$
5	$\sqrt{5} = \sqrt{6(5) - 5^2} = \sqrt{5}$

$2\sqrt{5} + 2 \cdot 3 + 2 \cdot \sqrt{5}$

$6 + 4\sqrt{5}$



a) Approximate the area under the graph of $g(x)$ from [0,6] using a Midpoint Riemann sum with 3 sub intervals of equal length

$\frac{6-0}{3} = \frac{6}{3}$

$6 + 4\sqrt{5}$

b) Express the area under the graph of $g(x)$ as a definite integral

$$\text{Area} = \int_0^6 \sqrt{6x - x^2}\, dx$$

c) Use technology to evaluate the integral.

$$\int_0^6 \sqrt{6x - x^2}\, dx = 14.137$$

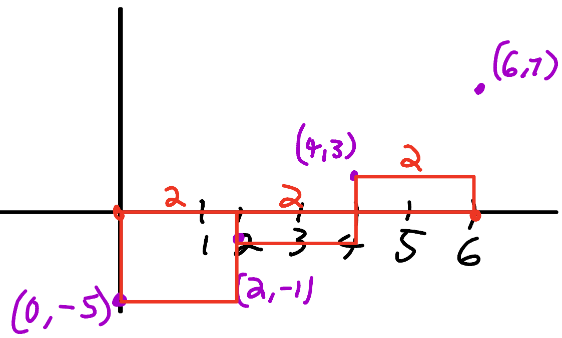
d) Confirm the answer to (c) using geometry

Semicircle $r = 3$

$$\frac{\pi r^2}{2} = \frac{\pi (3)^2}{2} = \frac{9\pi}{2} \qquad \frac{9\pi}{2} = 14.137$$

3. Approximate $\int_0^6 (2x-5)\,dx$ by partitioning the interval [0,6] into 3 subintervals each of length 2 and using a Left Riemann sum.

$\frac{6-0}{3}=2$



x	y
0	$-5 = 2(0)-5$
2	$-1 = 2(2)-5$
4	$3 = 2\cdot 4-5$
6	$7 = 2\cdot 6-5$

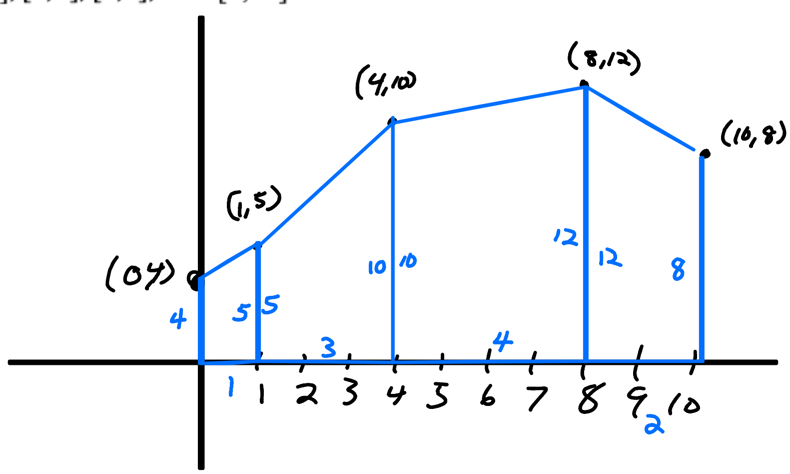
$2\cdot -5 + 2(-1) + 2\cdot 3$

$-10 - 2 + 6 = -6$

4. A function f is continuous on the closed interval [0,10] and has values

x	0	1	4	8	10
$f(x)$	4	5	10	12	8

Find an approximation to $\int_0^{10} f(x)\,dx$ using a trapezoidal sum with the four subintervals [0,1], [1,4], [4,8], *and* [8,10].



$\frac{1}{2}(4+5)\cdot 1 + \frac{1}{2}(5+10)\cdot 3 + \frac{1}{2}(10+12)\cdot 4 + \frac{1}{2}(12+8)\cdot 2$

$$\int \frac{e^x}{2+e^x}\,dx$$

$a = 2 + e^x$

$da = 0 + e^x dx$

$$\frac{da}{e^x} = \frac{\cancel{e^x}\,dx}{\cancel{e^x}}$$

$$\frac{da}{e^x} = dx$$

$$\int \frac{\cancel{e^x}}{a} \cdot \frac{da}{\cancel{e^x}}$$

$$\int \frac{da}{a} = \ln|a| + C$$

$$\ln|2+e^x| + C$$